

To Sell Or Rent?

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Abstract

A question that has been bugging me is - we own a house, if we want to move, should we rent or sell the house? Below I develop the logic for a calculator to determine which option is likely to bring us the biggest profit (or, these days, the smallest loss).

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1 A note on notation

This article is, in essence, a big algebra equation. I know, exciting, huh?

I have used the standard algebra notation I was taught back in the pleizoic to describe the algebra involved. I do however use a few moderately advanced algebra notations. The one most folks probably learned in school is $\sum$ which is called Sigma and is used for [summation](#). So, for example, $\sum_{x=1}^5 x = 1 + 2 + 3 + 4 + 5$.

I also use the standard algebraic subscript notation to represent functions of one argument. So $T_y = 1 + y$ would give the result $T_3 = 4$.

In a few cases I need functions that take more than one argument. To make reading the text easier I switch to using a standard parenthesis notation familiar to programmers. So $K(a, b) = a + b$ would result in $K(3, 5) = 8$. I choose not to use this notation for functions of a single argument because it can lead to confusion. Is $X(1+1)$ supposed to be a call to function X with the argument 2 or am I multiplying the value X by 2? By using subscript for single argument functions no confusion is possible. In the case of two or more argument functions the presence of the comma to separate the arguments makes the usage unambiguous.

Also, strictly as a convention, variables are lower case and functions/constants are upper case.

I also use a choice or case notation.

$$Y_x = \begin{cases} x = 0 & 10 \\ x \neq 0 & x \end{cases}$$

The case notation consists of the bracket with a series of mutually exclusive choices. Each choice consists of two parts, a condition and a consequent. If the condition is true then the consequent is the output of the function. The cases are generally written so that order should not matter. In other words if everything is written correctly then for any given set of arguments exactly one case should be true. If none are true then an error has occurred. Typically this means the function got an argument it wasn't supposed to. So in the previous case $Y_0=10$ while $Y_{40}=40$.

I also use exactly two symbols from [Boolean algebra](#). $\cap$ for the "and" operator and $\cup$ for the "or" operator.

I use the [modulus](#) (which I refer to as mod) operator and [quotient](#) operator to help me with dates. The modulus is the remainder of diving two numbers. So $5 \bmod 3 = 2$. The quotient, in this case, deals strictly with dividing two numbers and getting an integer result. Or put another way, the quotient is how many times one number can divide another number with no remainder. So $5 \text{ quotient } 3 = 1$.

The rest of the notation is bog standard algebra.

2 Things i'm really not sure about

Are expenses incurred in the sale of the house (transaction fees, agent fees, etc.) used to reduce the cost basis of the house or are they subtracted from rental income only? Right now I do not subtract them from the sales price of the house.

Let's say we have suspended losses and we sell the house after two years so we qualify for the \$500,000 tax exemption. Do we still need to take the suspended losses against the house (where they are essentially thrown away if the profit is \$500,000 or less) or do we take them against our income? For now I'm conservative and assume we have to take them against the house profit even though we are, in fact, throwing that money away by doing so.

In general I assume that suspended losses are first taken against any profit from the house and only then from any income from rent in the year we sold the house and finally against other income. I picked this order because it's the least advantageous so I figure that is what the government would do but I'm not sure.

If I sell our house mid-way through the year do I get to depreciate the property for the entire year or only the part of the year where we actually owned the house? Similarly if I buy a house (or start renting out the house) part way through the year do I get the full appreciation for the year or only partial appreciation?

In the year we sell the house do we deduct the expenses incurred in running the house first from the sales price of the house or from rental income (if any?). If we still have money left over (e.g. we didn't have any rental income or the house was sold at a loss) do we use it for the other category (e.g. if first used it for house profit then do we apply it to rental income? And visa versa)?

3 Opportunity cost and Inflation

Opportunity cost is a measure of how much it costs to take one financial choice versus another in monetary terms. In our case we want to know the opportunity cost of selling the house now versus selling the house later after having rented it out.

Imagine we could sell the house today for \$100. Or we could rent the house out and then sell it 5 years from now for a total profit (rent plus eventual sales price) of \$110. It's tempting to then argue "well, let's rent the house out!". But we have to take account of the opportunity cost of money. Is \$100 today worth the same, less or more than \$110 five years from now? Let's say, for example, that we would take the \$100 we could have made today and put it into a CD earning 5%. In that case the \$100 five years from now would be worth $\$100 * (1 + 0.05)^5 = \127 . So in fact we would, in this case, be better off selling the house now.

There is, however, another complication. Inflation. After all \$100 today isn't worth the same as \$110 five years from now in real terms (e.g. after adjusting for inflation). Let's say inflation is 3%. In that case \$110 five years from now is equal to $\$100 * (1 - 0.03)^5 = \94.46 . This also means that the value of the money we would get from the CD would be worth $\$127 * (1 - 0.03)^5 = \109.06 . So after accounting for inflation then renting out looks better.

All the previous numbers are obviously nonsense, they are just used as a simple example of the ideas involved.

4 Calculating if we should rent or sell (OpportunityCost)

The opportunity cost of renting versus selling is calculated as:

$$OpportunityCost = Rent - Sell$$

Rent The total profit or loss in real dollars if we rent the house out and then sell it

Sell The total profit or loss in real dollars if we sell the house immediately rather than rent it out

If the number is positive then we would be better off renting the house out and then selling it. If the number is negative then we would be better off selling the house now.

5 The total profit or loss in real dollars of selling the house now (Sell)

$$Sell = HouseSaleProfit_0 * (1 + RealMonthlyOpportunityCost)^{MonthsInBusiness}$$

HouseSaleProfit₀ the net profit from selling the house in month 0

RealMonthlyOpportunityCost is the monthly real (e.g. adjusted for inflation) opportunity cost of money (e.g. how much could we earn if we sold the house now and saved the money)

MonthsInBusiness is the months that we are in the business of renting out a house

Note that:

- I use a monthly opportunity cost in the calculations above
- I assume we will live in the house right up until we sell it.
- To facilitate easy comparisons I adjust the nominal result to a real result in current dollars by multiplying the $(1 - I)^{MonthsInBusiness}$ on the end.

6 The total profit or loss in real dollars of renting the house out and then selling it (Rent)

$$Rent = \left(\sum_{M=0}^{MonthsInBusiness-1} \frac{MonthlyRentalProfit_M}{(1 - MonthlyInflationRate)^M} (1 + RealMonthlyOpportunityCost)^{MonthsInBusiness-M} \right)$$

M represents a month we are in business

MonthlyInflationRate is the monthly inflation rate

MonthlyRentalProfit_M is the monthly nominal (e.g. not inflation adjusted) profit/loss for all sources (e.g. rent, selling the house, taxes, etc.) for month **M**.

Note that:

- I assume that all profit/loss is realized at the first day of the month so that we then account for inflation and opportunity cost for the whole month.
- I count months in business starting at 0. So if we are in business for 3 months then we will have **M** values of 0, 1 and 2.

So the previous equation takes the profit or loss for each month, adjusts it back to current dollars (our basis for comparison) and then accounts for the opportunity cost of investing the profit or losing the loss.

7 Months In Business

MonthsInBusiness measures how many months we will be in the business of renting our home out.

$$MonthsInBusiness = (MonthsToFindTenant + MonthsInLease) * LeaseCycles + MonthsToSell + 1$$

MonthsToFindTenant is the number of months it takes to find a tenant for the house

MonthsInLease is the number of months the lease lasts

LeaseCycles is how many lease cycles we plan to go through (e.g. how many leases will we sign before we decided to sell the house)

MonthsToSell is how many months it takes to sell the house

The one added on the end accounts for the month in which the house is actually sold.

8 Nominal profit/loss from selling the house (HouseSaleProfit)

$$\begin{aligned} HouseSaleProfit_m &= HouseSalesPrice_m - HouseSaleExpenses_m \\ HouseSaleExpenses_m &= HouseSalesPrice_m * ExciseTax + BuyingAgentFee_m + SellingAgentFee_m + OtherSalesFees_m + RemainingLoanBalance_m \end{aligned}$$

HouseSalesPrice_m is the sales price of the house in nominal dollars in month m

ExciseTax is the percentage of the sales price of the house charged as excise tax

BuyingAgentFee_m is the fee for the buying agent in nominal dollars in month m

SellingAgentFee_m is the fee for the selling agent in nominal dollars in month m

OtherSalesFees_m is random other fees in nominal dollars (i.e. recording fees, title fees, etc.) in month m

RemainingLoanBalance_m is the principal remaining on the mortgage of the house in nominal dollars on the first day of month m

As previously mentioned the assumption is that the house sells on the first day of month. Also note that we live in Washington State where the seller typically pays all the agent fees.

$$\begin{aligned} Inflate(init, m) &= init * (1 + MonthlyInflationRate)^m \\ HouseSalesPrice_m &= Inflate(HouseSalesPrice_0, m) \\ AgentFee(feeType, fee, m) &= \begin{cases} feeType = Percentage & HouseSalesPrice_m * fee \\ feeType = FlatFee & Inflate(fee, m) \end{cases} \\ BuyingAgentFee_m &= AgentFee(BuyingAgentFeeType, BuyingAgentFeeNumber, m) \\ SellingAgentFee_m &= AgentFee(SellingAgentFeeType, SellingAgentFeeNumber, m) \\ OtherSalesFees_m &= Inflate(OtherSalesFees_0, m) \end{aligned}$$

Inflate is just a handy function to take an initial value and inflate it using the monthly inflation rate for a specified number of months.

We calculate the house sales price for a given month as increasing at the rate of inflation. Historical, in most areas, that's typically the rate that house prices go up. Of course recently they have only been going down. So for now I'll just stick with inflation.

These days agent fees come in two flavors. There is the typical percentage fee and then there are firms that charge flat fees. I set up the model so it can handle either situation. HouseSalesPrice₀, BuyersAgentFeeNumber, SellingAgentFeeNumber, OtherSalesFees₀ are all the values of the various entities in month 0 and are assumed to be provided to the calculation along with BuyersAgentFeeType and SellingAgentFeeType.

9 Principal remaining on mortgage at the start of the month (RemainingLoanBalance)

The RemainingLoanBalance and MonthlyLoanPayment equations are taken from the [Mortgage Professor's website](#).

$$RemainingLoanBalance'_m = \begin{cases} m < MonthsInLoan & \frac{OriginalLoanAmount[(1+MonthlyLoanInterest)^{MonthsInLoan} - (1+MonthlyLoanInterest)^m]}{(1+MonthlyLoanInterest)^{MonthsInLoan} - 1} \\ m \geq MonthsInLoan & 0 \end{cases}$$

m is the month of the loan, we start counting from 0

OriginalLoanAmount is the original loan amount (in nominal dollars) (e.g. if we got a \$500 loan then this value would be \$500)

MonthsInLoan is the total number of months in the loan (e.g. a 30 year loan would have $30 \times 12 = 360$ payment periods)

MonthlyLoanInterest is the yearly interest rate on the mortgage loan divided by 12 (assuming the mortgage is paid monthly)

$$RemainingLoanBalance_m = RemainingLoanBalance'_{m+LoanMonthAtStart}$$

LoanMonthAtStart is the month the loan is in when we start our rental business (e.g. if we had the loan for 20 months and the next month we entered the rental business then this value would be 21)

$$MonthlyInterestPaid_m = \begin{cases} MonthlyPrincipalPaid_m > 0 & MonthlyLoanPayment - MonthlyPrincipalPaid_m \\ MonthlyPrincipalPaid_m = 0 & 0 \end{cases}$$

$$MonthlyPrincipalPaid_m = |RemainingLoanBalance_{m+1} - RemainingLoanBalance_m|$$

$$MonthlyLoanPayment = \frac{OriginalLoanAmount * \left[MonthlyLoanInterest * (1 + MonthlyLoanInterest)^{MonthsInLoan} \right]}{(1 + MonthlyLoanInterest)^{MonthsInLoan} - 1}$$

10 Monthly Profit/Loss From Renting (MonthlyRentalProfit)

Each month we are in the business of renting the house out we make or lose a certain amount of money. This amount is calculated by:

$$MonthlyRentalProfit_m = RentalIncome_m + HouseSaleIncome_m - TaxDeductibleExpenses_m - NonTaxDeductibleExpenses_m$$

RentalIncome_m is the nominal rental income earned in month M (this value will be zero for months where the house isn't leased out)

HouseSaleIncome_m is the nominal income we earned from selling the house in month M (this would have to be month MIB+1, otherwise this value is 0)

TaxDeductibleExpenses_m is the nominal tax deductible expenses incurred in month M

NonTaxDeductibleExpenses_m is the nominal non-tax deductible expenses incurred in month M

The reason for splitting up the costs this way is due to taxes. As will be defined later the tax handling for profit from rental income versus selling the house is handled differently from each other. In addition, for calculating taxes, we need to know which expenses are tax deductible and which are not.

Note that:

- When we do the final accounting we will stick the federal income tax bill into Non-Tax-Deductible-Expenses_m.
- Since we live in Washington State, which doesn't have a state income tax (yet), we don't take state income tax into consideration.

11 House State (HouseState)

To make life a bit easier later on I'm going to introduce now a variable called HouseState. It represents the state of the house in a certain month. The available states are: for lease, leased, for sale and sold.

$$\begin{aligned}
 HouseState_m &= \begin{cases} m < MonthsActivelyRenting & \begin{cases} MonthInRentalCycle_m < MonthsToFindTenant & ForLease \\ MonthInRentalCycle_m \geq MonthsToFindTenant & Leased \end{cases} \\ m \geq MonthsActivelyRenting & \begin{cases} m < MonthsInBusiness - 1 & ForSale \\ m = MonthsInBusiness - 1 & Sold \end{cases} \end{cases} \\
 MonthsActivelyRenting &= (MonthsToFindTenant + MonthsInLease) * LeaseCycles \\
 MonthInRentalCycle_m &= m \bmod (MonthsToFindTenant + MonthsInLease)
 \end{aligned}$$

Keep in mind that m, the month, counts from 0.

12 Nominal Rental Income (RentalIncome)

$$\begin{aligned}
 RentalIncome_m &= \begin{cases} HouseState_m = Leased & MonthlyRent_m \\ HouseState_m \neq Leased & 0 \end{cases} \\
 MonthlyRent_m &= Inflate(MonthlyRent_0, (m \text{ quotient } RentalCycleLength) * RentalCycleLength) \\
 RentalCycleLength &= MonthsToFindTenant + MonthsInLease
 \end{aligned}$$

The previous assumes that the rent is set at the start of each lease cycle when we start looking for a tenant. It also assumes that the rent will go up only by the rate of inflation up to the start of the lease cycle. For example, let's say that we assume it takes one month to find a tenant and that a lease lasts for one month. We are now in month 3 (remember, we start counting from 0). So $(3 \text{ quotient } (1+1))(1+1) = 2$. So the value of the rent is increased by 2 months. Which makes sense because the rent was set at the start of the lease cycle which was in month 2 which means only months 0 and 1 have fully passed, hence 2 months of inflation.

13 Nominal House Sale Income (HouseSaleIncome)

$$HouseSaleIncome_m = \begin{cases} HouseState_m = Sold & HouseSalesPrice_{m - MonthsToSell} \\ HouseState_m \neq Sold & 0 \end{cases}$$

The previous assumes that the sales price of the house is set at the start of the period when we try to sell the house. It also assumes that the house only increases in value at the rate of monthly inflation.

14 Nominal Tax Deductible Expenses (TaxDeductibleExpenses)

$$\begin{aligned}
 TaxDeductibleExpenses_m &= \begin{cases} HouseState_m = ForLease & \begin{cases} MonthInRentalCycle_m = 0 & FixedMonthlyCost_m + PrepHouse_m \\ MonthInRentalCycle_m > 0 & FixedMonthlyCost_m \end{cases} \\ HouseState_m = Leased & \begin{cases} MonthInRentalCycle_m = MonthsToFindTenant & FixedMonthlyCost_m + \\ & RentalIncome_m * (TenantFindingFee + ManagementFee) \\ MonthInRentalCycle_m > MonthsToFindTenant & FixedMonthlyCost_m + \\ & RentalIncome_m * ManagementFee \end{cases} \\ HouseState_m = ForSale & \begin{cases} m = MonthsActivelyRenting & FixedMonthlyCost_m + PrepHouse_m \\ m > MonthsActivelyRenting & FixedMonthlyCost_m \end{cases} \\ HouseState_m = Sold & HouseSaleExpenses_m - RemainingLoanBalance_m \end{cases} \\
 PrepHouse_m &= Inflate(PrepHouse_0, m)
 \end{aligned}$$

FixedMonthlyCost_m is the fixed nominal monthly cost of owning the house in month M

PrepHouse_m is the nominal cost of preparing the house for a new tenant (or buyer) in month M

TenantFindingFee is the management tenant finding fee, this is a multiple of months of rent (i.e. agencies will often charge the equivalent of one month's rent to find a tenant)

ManagementFee is the management fee charged by the company that manages our home, this value is expected to be a percentage of the monthly rent (i.e. agencies in our area tend to charge about 10% of each month's rent to manage the home)

Every month we are in the business of renting out the house but the month we actually sell the house we have to pay a set of fixed monthly costs. These include things like the interest part of the mortgage payments, property taxes, maintenance, etc. I'll detail all of these in a subsequent section. These expenses are all tax deductible.

In certain months however we have additional expenses. For example, the first month we start looking for a new tenant we have to clean up the house. The same thing for when we prepare the house for sale. We call these expenses PrepHouse and will detail them later.

Marina and I don't have the intestinal fortitude to find and manage tenants. So if we are going to rent the house out we are going to get a management company to help us out. Management companies typically charge a multiple of the monthly rent (typically one months rent) to find tenants. We book that fee the first month a lease starts. In addition every month the house is leased out we have to pay a management fee to manage the unit which is also usually a multiple of the rent.

The month that the house is sold we don't have to pay the fixed monthly cost because that is usually paid for by the buyer. But we do have to pay for all the various fees related to selling the house. Notice that the HouseSaleExpenses function also includes the remaining loan balance. But for tax purposes we will count that as a non-tax deductible expense. So we have to compensate here for that value in HouseSaleExpenses by taking it out. If we don't we will include the remaining loan balance as both a tax deductible and non-tax deductible expense.

14.1 Fixed Monthly Tax Deductible Costs of owning a house (FixedMonthlyCost)

$$FixedMonthlyCost_m = MonthlyInterestPaid_m + PropertyTaxes_m + PropertyLiabilityInsurance_m + AssociationDues_m + GardenMaintenance_m + HomeMaintenance_m$$

MonthlyInterestPaid_m is the nominal mortgage interest, if any, due that month, this has been previously defined in Section 9

PropertyTaxes_m is the property taxes due that month

PropertyLiabilityInsurance_m is the property and liability insurance prorated for that month

AssociateDues_m are the association dues prorated for that month

GardenMaintenance_m are the garden maintenance fees due that month (since we assume the tenant will not handle any garden upkeep beyond watering the lawn)

HomeMaintenance_m are the costs of house maintenance prorated for the month.

$$PropertyTaxes_m = HouseSalesPrice_{(m \text{ quotient } 12) * 12} * MonthlyPropertyTaxRate$$

MonthlyPropertyTaxRate is the percentage of the assessed value of the house that is paid out each month as property tax. This model assumes that the property tax percentage is a constant over time which it isn't but in most areas it typically raises fairly slowly, often by law.

Property tax is measured as a percentage of the house's assessed value. I use the sales price of the house as the estimated value. This probably isn't as wacky as it sounds since of late our assessments have both over and undershot the actual value of the house. But I set things up so that we take the 'assessed' value of the house every 12 months (arbitrarily starting when we go into 'business', although really it should be aligned with the assessment calendar) and then divide that value over 12 months into a monthly 'cost'. In reality we pay property tax twice a year but for modeling purposes I'll treat it as a monthly expense.

$$\begin{aligned} PropertyLiabilityInsurance_m &= Inflate(PropertyLiabilityInsurance_0, (m \text{ quotient } 12) * 12) \\ AssociationDues_m &= Inflate(AssociationDues_0, (m \text{ quotient } 12) * 12) \\ GardenMaintenance_m &= Inflate(GardenMaintenance_0, (m \text{ quotient } 12) * 12) \end{aligned}$$

The three previous values all follow the same pattern. They each start off with some base value (their 0 month value which is assumed to be a constant provided to the system) that represents a monthly pro-rated cost of the service when the business begins. This cost is assumed to not change until 12 months have passed at which point the cost is re-adjusted for inflation. The reason for this approach is that we typically sign year long contracts for these services.

$$HomeMaintenance_m = Inflate(HomeMaintenance_0, m)$$

As with the previous values house maintenance has a starting monthly value that is provided as a constant to the system. But unlike the others the monthly payments I let this one adjust monthly since the various random house maintenance costs are not paid on contract but as they happen.

15 Nominal Non-Tax Deductible Expenses (NonTaxDeductibleExpenses)

$$NonTaxDeductibleExpenses_m = \begin{cases} HouseState_m \neq Sold & MonthlyPrincipalPaid_m + Taxes_m \\ HouseState_m = Sold & RemainingLoanBalance_m + Taxes_m \end{cases}$$

In all the months except for the one in which the house is sold the non-tax deductible expenses consist of the principal (if any) due on the mortgage and the taxes due that month. In the month the house is sold no mortgage payments are due (since the house is being paid off) but rather the entire remaining principal is due plus taxes.

16 Taxes (Taxes)

$$\begin{aligned} YearlyTaxes_y &= YearlyIncomeTaxes_y + HouseCapitalGains_y \\ Taxes_m &= \frac{YearlyTaxes_{FindYear_m}}{BusinessMonthsInYear_{FindYear_m}} \end{aligned}$$

YearlyTaxes_y is the total taxes due to the federal government in year y.

Taxes_m is the total taxes due to the federal government in month m.

BusinessMonthsInYear_y are the total number of months we are in business during year y.

YearlyIncomeTaxes_y is the total taxes paid to the federal government due to income such as our salaries, short term capital gains and income from renting out the house in year y.

HouseCapitalGains_y is the total taxes paid to the federal government from the sale of our rental property

FindYear_m is the year, counting from 0 as the year when we start in business, that the month falls in

I only worry about the taxes we have to pay specifically for our rental activities. But to calculate those taxes, given the progressive Federal tax system, I first have to know all of our income for the year. That way I can figure out the part that was incurred due to our rental activities. Then I take the 'extra' taxes due each year and break them up into monthly payments. The idea being that each month we pay exactly the portion of our tax due from expenses/profits accrued in that month. In reality we would pay at the end of the year and take precautions against underpaying but I generally don't expect these tricks to earn me enough money to be worth accounting for. So to simplify life I just account for taxes in each month due.

One complication is that I don't treat the month we actually sell the house (remember, that's the month where the house is sold on the very first day) as a business month. Now imagine that the month the house sells is January. This means the last business month was the previous December. Which means when we ask BusinessMonthsInYear for YearSold we will get 0 because we weren't in business in the year the house sold. This contrivance on my part is a mistake and someday I should re-think the system without it but for now it's there and I deal with this case by putting in extra logic in Taxes to look for it and properly compensate.

$$\begin{aligned}
 FindYear_m &= (m + FirstMonthOfRental) \text{ quotient } 12 \\
 BusinessMonthsInYear_y &= \begin{cases} y = 0 & \begin{cases} FirstMonthOfRental + MonthsInBusiness - 1 < 12 & MonthsInBusiness \\ FirstMonthOfRental + MonthsInBusiness - 1 \geq 12 & 12 - FirstMonthOfRental \end{cases} \\ y > 0 \cap y < YearSold & 12 \\ y = YearSold & MonthsInBusiness - \sum_{c=0}^{YearSold-1} BusinessMonthsInYear_c \end{cases} \\
 YearSold &= FindYear_{MonthsInBusiness-1}
 \end{aligned}$$

FirstMonthOfRental is the month of the year when we first started in the rental business with January equal to 0

YearSold the year in which the rental property is sold

17 Yearly Income Taxes (YearlyIncomeTaxes)

$$YearlyIncomeTaxes_y = FederalTaxDue(y, NetWorkIncome_y + ZeroOrPositive_{RawRentalIncome_y}) - FederalTaxDue(y, OriginalWorkIncome_y)$$

$$FederalTaxDue(y, a) = \sum_{tb=0}^{TotalTaxBrackets} TaxBracket(tb, y, a)$$

$$TaxBracket(tb, y, a) = \begin{cases} tb \neq 0 \cap a \leq TaxBracketMax(tb-1, y) & 0 \\ tb \neq 0 \cap a > TaxBracketMax(tb-1, y) \cap a \leq TaxBracketMax(tb, y) & TaxBracketRate_{tb} * (a - TaxBracketMax(tb-1, y)) \\ tb \neq 0 \cap a > TaxBracketMax(tb, y) & TaxBracketRate_{tb} * (TaxBracketMax(tb, y) - TaxBracketMax(tb-1, y)) \\ a > TaxBracketMax(tb, y) & TaxBracketRate_{tb} * TaxBracketMax(tb, y) \end{cases}$$

$$TaxBracketMax(tb, y) = Inflate(TaxBracketMaxTable(tb), y * 12)$$

FederalTaxDue(y,a) is the total tax due for year y on amount a

TaxBracket(tb,y,a) is the total tax due for the tax bracket tb on amount a during year y

TotalTaxBrackets is the total number of Federal income tax brackets starting at 0

TaxBracketMax(tb,y) is the dollar value of the top of the tax bracket tb during year y

TaxBracketRate_{tb} is the percentage of the dollars in the bracket that have to be paid to taxes

NetWorkIncome_y is the income derived from sources other than the rental property but adjusted for any write offs the rental property provides

RawRentalIncome_y is the income derived from the rental property adjusted for rental property related tax deductions

OriginalWorkIncome_y is the income that would have been taxed if we hadn't gotten into the rental business in year y

In theory it is quite possible for YearlyIncomeTaxes to be a negative number. This means that all of our deductions exceed our income. This is called a Net Operation Loss and has its own tax rules. If I see a NOL show up I'm probably going to talk to a tax attorney (well, actually I'm going to probably do that anyway) because the rules for a NOL are more than my brain can handle at the moment.

The previous assumes that the tax rate for each bracket as well as the number of brackets will remain fixed. What will change, however, is the nominal dollar value of the brackets which will be adjusted for inflation each year. The following table is an example of TaxBracketMaxTable and TaxBracketRate values for 2010 for married couples filing jointly. assuming 2010 is year 0.

tb	TaxBracketMaxTable(tb,0)	TaxBracketRate _{tb}
0	\$16,749	0.1
1	\$67,999	0.15
2	\$137,299	0.25
3	\$209,249	0.28
4	\$373,649	0.33
5	infinity	0.35

18 Net Work Income (NetWorkIncome) and Original Work Income (OriginalWorkIncome)

$$\begin{aligned} NetWorkIncome_y &= OriginalWorkIncome_y + \begin{cases} y = YearSold & ZeroOrNegative_{RawRentalIncome_y} \\ y \neq YearSold & 0 \end{cases} \\ OriginalWorkIncome_y &= Salary_y - StandardDeduct_y \\ Salary_y &= Salary_0 * (1 + 12 * MonthlyInflationRate + YearlySalaryIncrease)^y \\ StandardDeduct_y &= Inflate(StandardDeduct_0, Y * 12) \end{aligned}$$

Salary_y is the amount of salary and other workd income we expect to make in year y.

StandDeduct_y is the standard deduction in year 0, we assume that over time it grows with the rate of inflation.

YearlySalaryIncrease the percentage we believe our salaries will go up in real (e.g. inflation adjusted) terms each year

I have broken up the calculation of our income taxes into two parts. The first part, NetWorkIncome, is the taxes we owe on our non-rental income. Normally this would just be whatever we made minus the standard deduction. Yes, we own a mortgage so we itemize but the difference isn't enormous and I'd like to finish this calculator some day so I'm not going to build a fully accurate income tax calculator. So I use the standard deduction as a stand in for all of our other deductions. I'm also doing my best to ignore the alternative minimum tax.

This all simplifies things nicely with one exception, suspended losses.

Near as I can tell the IRS has decided (for what seem like good reasons) that deductible expenses spent on the rental house can only be deducted from income earned renting out the house. Which means that if our deductions are too large we just have to 'bank' them for a future day. In other words if our expenses in year 0 for renting out the house were \$100 and our income was \$10 then we can fully deduct against the \$10 income and not pay taxes on it but we have \$90 of unused deduction left over which we can't use against anything. This \$90 is called a suspend loss. The loss still exists but we can't use it right now. All we can do is bank it for the future.

There are three ways we can use suspended losses:

1. We can deduct it from future rental income. For example, imagine in year 1 we have rental expenses of \$100 but rental income of \$200. In that case we can deduct all the expenses against the income but we still have \$100 in income left over. We can now bring up the \$90 from year 0 and use it so the amount of rental income liable to income tax is only \$10.
2. Some folks can deduct up to \$25,000 of income tax loses against their regular income each year. However Marina and I don't qualify for that deduction so I'm not going to worry about it.
3. When we sell the rental property (assuming we don't own any other rental property, in which case things get icky) any unused suspended losses can be taken directly against our other income.

Option 3 is where SuspendedLoses comes from. It represents any suspended loses we couldn't exhaust on the rental property and can now, after the house is sold, use against our other income. This means that the only time SuspendedLoses won't be set to 0 is in the year we sell the house.

19 Raw Rental Income (**RawRentalIncome**)

$$\begin{aligned}
 \text{RawRentalIncome}_y &= \text{SingleYearRentalIncome}_y + \begin{cases} y = 0 & 0 \\ y \neq \text{YearSold} & \text{ZeroOrNegative}_{\text{RawRentalIncome}_{y-1}} \\ y = \text{YearSold} & \text{ZeroOrNegative}_{\text{HouseSaleTaxBasis}} \end{cases} \\
 \text{SingleYearRentalIncome}_y &= \left(\sum_{m=\text{FirstMonth}_y}^{\text{LastMonth}_y} (\text{RentalIncome}_m - \text{TaxDeductibleExpenses}_m) \right) - \text{HouseDepreciation}_y \\
 \text{HouseDepreciation}_y &= \begin{cases} y > 27 & 0 \\ y < 27 & \frac{\text{HouseSalesPrice}_0}{27.5} \\ y = 27 & \frac{0.5 * \text{HouseSalesPrice}_0}{27.5} \end{cases} \\
 \text{FirstMonth}_y &= \begin{cases} y = 0 & 0 \\ y > 0 & (12 - \text{FirstMonthOfRental}) + ((y - 1) * 12) \end{cases} \\
 \text{LastMonth}_y &= \left(\sum_{c=0}^y \text{BusinessMonthsInYear}_c \right) - 1
 \end{aligned}$$

RawRentalIncome_y is the total profit or loss made on rental activities for year y .

SingleYearRentalIncome_y provides the net rental income generated within a single year, it doesn't account for past suspended losses

HouseDepreciation_y The total amount the house can be depreciated in the specified year.

FirstMonth_y is the business month that matches the first calendar month we are in business in the specified year. So, for example, in year 0 we always start in business month 0.

LastMonth_y is the business month we are in during the last month of the specified year that we are actually in business.

The total income subject to income tax each year equals to the rental income minus rental deductions. Deductions in this case come in two forms. There are various business expenses which `TaxDeductibleExpenses` already covers. Then there is depreciating the value of the house. The IRS allows land lords to depreciate the value of their rental properties over 27.5 years. I have no idea why it's 27.5 years but there you go. The actual depreciation isn't exactly as given in `HouseDepreciation`, it's apparently a little different in the first and last year, but it's close enough for now. The depreciation is based on the value of the property at the time I started trying to rent it. How to calculate this value is a fairly black art but I'll take a wild guess at it by using the sales value of the property at the time I start renting. Note that the depreciation is not adjusted for inflation.

20 Capital Gains Due on House Sale (HouseCapitalGains)

$$\begin{aligned}
 \text{HouseCapitalGains}_y &= \begin{cases} y \neq \text{YearSold} & 0 \\ y = \text{YearSold} & \begin{cases} \text{TaxableGainOnHouseSale} \leq \text{TotalHouseDepreciation} & \text{TaxableGainOnHouseSale} * 0.25 \\ \text{TaxableGainOnHouseSale} > \text{TotalHouseDepreciation} & (\text{TotalHouseDepreciation} * 0.25) + \\ & ((\text{TaxableGainOnHouseSale} - \text{TotalHouseDepreciation}) * 0.15) \end{cases} \end{cases} \\
 \text{TaxableGainOnHouseSale} &= \begin{cases} \text{HouseSaleTaxBasis} \leq 0 & 0 \\ \text{HouseSaleTaxBasis} > 0 & \begin{cases} \text{YearSold} < 3 & \begin{cases} \text{HouseSaleTaxBasis} \leq 500000 & 0 \\ \text{HouseSaleTaxBasis} > 500000 & \text{HouseSaleTaxBasis} - 500000 \end{cases} \\ \text{YearSold} \geq 3 & \text{HouseSaleTaxBasis} \end{cases} \end{cases} \\
 \text{HouseSaleTaxBasis} &= \text{HouseSalesPrice}_{\text{MonthsInBusiness} - \text{MonthsToSell} - 1} - \text{HouseSalesPrice}_0 + \\
 &\quad \text{TotalHouseDepreciation} + \text{ZeroOrNegative} \left(\text{SingleYearRentalIncome}_{\text{YearSold}} + \begin{cases} \text{YearSold} = 0 & 0 \\ \text{YearSold} > 0 & \text{RawRentalIncome}_{\text{YearSold} - 1} \end{cases} \right) \\
 \text{TotalHouseDepreciation} &= \sum_{c=0}^{\text{FindYear}_{\text{MonthsInBusiness} - 2}} \text{HouseDepreciation}_c
 \end{aligned}$$

TaxableGainOnHouseSale_y is the total tax adjusted gain on the property sale

TotalHouseDepreciation is the total housing deduction we should have taken every year up to the current year

HighRate_a is the amount of tax due using the higher 0.25% rate due on profit up to the amount previously taken as a housing depreciation

LowRate_a is the amount of tax due once the high rate is paid off

When we sell the rental property we have to calculate how much in taxes we owe. To determine this we need to know our tax basis, that is, how much of the profit (loss will be dealt with later) is subject to taxation. The basis of the calculation is to take how much we sold the house for and then subtract the original tax basis which was the amount we used to calculate the depreciation (e.g. the sales price of the house when we started out). But the IRS also requires that we 'give back' the depreciation deductions we took. So we have to increase the tax basis by an amount equal to all of the housing depreciation deductions we took. Finally if we have any suspended losses from the previous year we need to decrease the tax basis to account for those.

If the end result is negative then this negative number just becomes yet another suspended loss that we can try to take against our rental income and our work income.

If the result is positive then we may have to pay taxes. This is where the two out of five year rule comes into effect. This rule states (more or less) that if we sell a house that we owned and lived in during two out of the last five years then up to \$250,000 of profit as an individual for \$500,000 of profit as a married couple is exempt from tax. The law was actually changed in the Housing and Economic Recovery Act of 2008 but those changes appears to apply to people who first rent out a property and then live in it. In our case we first have lived in it and are then renting it out so it looks like we still qualify for the full deduction.

If we do owe any taxes those taxes are capital gains, not income tax. But they appear to be special capital gains. Any money we made up to the amount of the housing depreciation amount is taxed at 25% and any money after that is taxed at 15%.

21 Utility functions

$$\begin{aligned} ZeroOrNegative_n &= \begin{cases} n > 0 & 0 \\ n \leq 0 & n \end{cases} \\ ZeroOrPositive_n &= \begin{cases} n > 0 & n \\ n \leq 0 & 0 \end{cases} \end{aligned}$$

There are just utility functions to make the equations smaller.